



Better Methods. Better Outcomes.

# Webinar Series

## Transferability: How to Make It Work Best for Your Model

Date:

February 19, 2015

Speakers:

Thomas Rossi

(Cambridge Systematics)

Chandra Bhat

(Center for Transportation  
Research, Univ. of Texas-  
Austin)



## DISCLAIMER

*The views and opinions expressed during this webinar are those of the presenters and do not represent the official policy or position of FHWA and **do not constitute an endorsement, recommendation or specification by FHWA.** The webinar is based solely on the professional opinions and experience of the presenters and is made available for information and experience sharing purposes only.*

## Administrative Items

- **The** session will be recorded. The recorded webinar is available after the session via GovDelivery.
- **All** participant phone lines are muted.
- **Please** answer the polls to help us improve future webinars.
- **This** webinar will last approximately one hour. We could have additional 30 minutes for more questions and answers if needed.
- **A** Q&A pod window is displayed on your screen and you can enter your questions there anytime. The presenters will answer them during the Q&A session.
- **The** webinar is being live close-captioned for the hearing impaired.

# Acknowledgments

## Guide for Travel Model Transfer

- Managed by Sarah Sun, Federal Highway Administration
- Reviewers:
  - Jeremy Raw (FHWA)
  - Eric Pihl (FHWA)
  - Supin Yoder (FHWA)
  - Elaine Murakami (FHWA)
  - Brian Gardner (FHWA)
  - Ken Cervenka (FTA)

# Webinar Agenda

- Summary of the transferability issue
- Guidelines for model transfer
  - Data sufficiency for model estimation and updating
  - Variable Specification
  - Choosing the Estimation Context
  - Model transfer methods
- Examples of model transfer

## Introduction to Model Transfer – Model Parameters

- Mathematical relationships in travel models depend on *parameters*
- We do not know the “true” values of model parameters
- So we use *estimates* of the values of model parameters

## Introduction to Model Transfer – Model Parameters

- Model estimation is performed using data such as travel survey results
- We do not always have sufficient data for model estimation
- So sometimes we transfer models from other contexts

## Introduction to Model Transfer – Model Parameters

- Parameters within a model are *interdependent*
- The *correct way* to transfer model parameters is to transfer the *entire model*
- Partial transfer is risky, as parameter estimates can be affected by the presence of other variables in the model



## Introduction to Model Transfer – Concepts

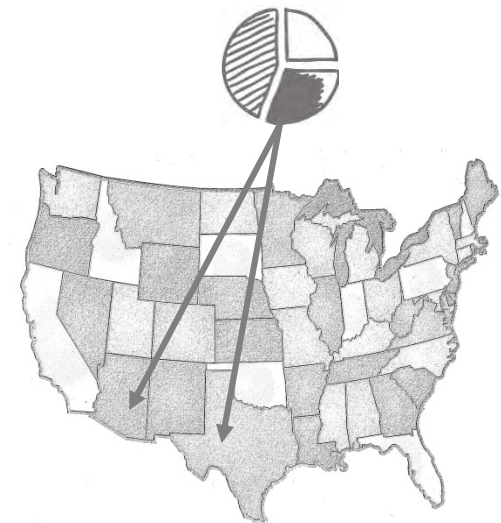
- *Spatial transfer* – Transfer of model from one geographic location to another
- *Estimation context* – Where the model to be transferred was estimated
- *Application context* – Where it is desired to transfer the model to

# Model Components

- Amenable to model transfer
  - Trip production
  - Trip attraction
  - Mode choice
  - Time of day
  - Vehicle availability
- Unlikely to have a true estimation context:
  - Highway and transit assignment
- Area-dependent parameters
  - Trip distribution

## Research Summary

- A literature review of research into spatial transferability of models was performed.
- Discussion in the context of:
  - The trip-based modeling approach
  - The activity-based approach



## Research Summary

The synthesis of transferability studies in the context of *trip-based models*:

- Provides mixed results regarding the effectiveness and validity of transfer.
- Indicates that the extent of transferability improves with better variable specification and a disaggregate-level model



## Research Summary

The synthesis of transferability studies in the context of *trip-based models*:

- Emphasizes that, whenever possible, some level of model updating is desirable using local data from the application context
- Suggests that even simple updating procedures such as a constants-only updating scheme using aggregate travel data in the application context provide superior transferability than the simple (no-update) transfer approach.

## Research Summary

The synthesis of transferability studies in the context of *activity-based models (ABMs)*:

- Improved ABM behavioral basis does seem to manifest itself in the form of improved transferability potential, especially in those components that are not associated with travel mode and location choices.



## Research Summary

The synthesis of transferability studies in the context of *activity-based models (ABMs)*:

- There is more consistency in the transferability results of the limited number of ABM transferability studies undertaken thus far than in the vast body of transferability literature on trip-based model components

(Whether this is simply a chance occurrence in the limited ABM studies or a true improvement in transferability because of the improved behavioral basis of ABMs remains to be seen).



## Research Summary

The synthesis of transferability studies in the context of *activity-based models (ABMs)*:

- It is important to undertake more ABM transfer studies with a more diverse set of regions than the set of regions used so far.

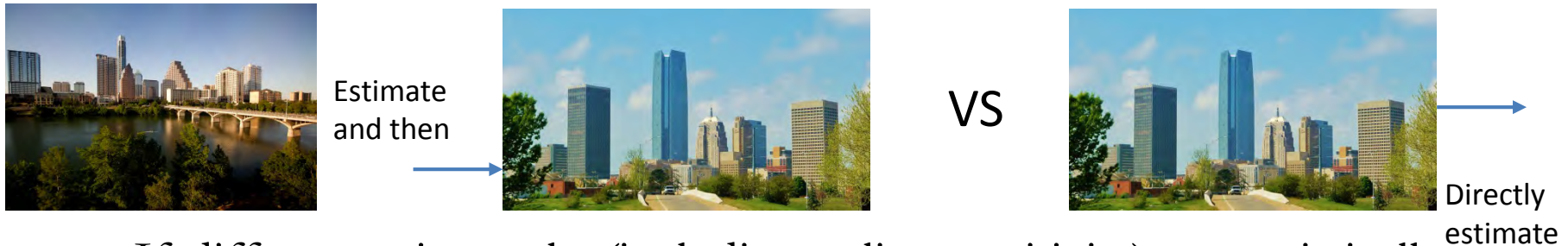
Note: The updating methods used in current ABM transfer studies have been the simple transfer approach and the constants updating approach.





## When to Transfer Models?

- How to demonstrate that transfer is okay?
- Estimate a model using survey data from Region A, apply the model to input data for Region B, and compare the results to those from a model estimated using survey data from Region B.



- If differences in results (including policy sensitivity) are statistically insignificant, transfer may be valid.
- Such a transferability study is unlikely to be undertaken by a planning agency since it would require more effort than simply estimating models from local data, and there is no need for model transfer when sufficient local estimation data exist.

# When to Transfer Models?

- Some evidence that model transfer may be more successful if estimation and application contexts are “similar” but...  
... there has been no systematic effort to quantify what constitutes “similarity”.
- Would be beneficial to understand which model components are more (or less) transferable, so agencies might target data collection for dimensions where local data will significantly improve representations of travel behavior.
- If there is no local data, no choice but to transfer a model.
- If there is some data for the application context, but not enough to perform full model estimation, there is evidence that using the local data to update the models from the estimation context can substantially improve the model transfer.

# Data Sufficiency

*(for Model Estimation and Updating)*

- It is impossible to develop a checklist for the sufficiency of model estimation data for various types of models
- Not only total sample size, but also samples by segment are critical
- It would be desirable for model estimation, application, and validation considerations to dictate survey sample sizes, but...
  - Budget constraints often limit survey sample sizes
  - Other considerations can affect sample sizes and segmentation

## Variable Specification

- No single set of variables that is universally used for a particular model component
- If a variable is defined differently between the estimation and application contexts, it is not correct to transfer the parameter(s) associated with the variable

## Variable Specification

- Even when a variable is defined identically in the application and estimation contexts, the associated parameters may not be transferable if the model specifications are not the same

Consider:

$$Y = B_0 + B_1X_1 + B_2X_2 + B_3X_3$$

Say that variable  $X_3$  is not available in the application context...

Then...

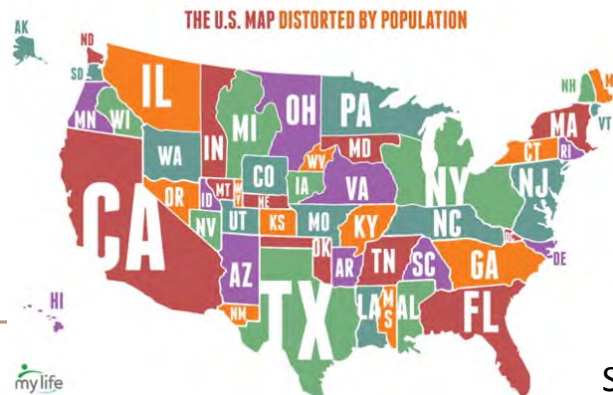
*Parameter estimates  $B_0$ ,  $B_1$ , and  $B_2$  are affected by the absence of  $X_3$*

## Choosing the Estimation Context

- If a model is to be transferred, it makes sense to choose the best available estimation context—that is, the context from which the model transfer is most likely to be valid.
- Note that many of the studies cited used data from only two contexts, and so conclusions about the quality of different types of estimation contexts were not possible in these cases.
- The relatively small amount of information provided in the literature does not provide substantial guidance on the selection of the estimation context for transferring models.

## Choosing the Estimation Context

- However, the following conclusions may be drawn from the literature review:
  - There is some evidence that transferability is enhanced when the estimation context is more “similar” to the application context, in terms of area population/size and socioeconomic make-up.
  - But “similarity” has not been rigorously defined.
  - Within the U.S., there is evidence that transferability is enhanced if the estimation context is in the same state as the application context, at least for relatively larger states.



## Model Transfer Methods

### *Simple Transfer*

- The model from the estimation context is simply used “as is,” and parameters may be revised during model calibration.
- No updating of any kind is made to the estimation context parameters when applied in the application context.



## Model Transfer Methods

### *Transfer Scaling*

- The application context utility function scales and constants are estimated from a small application context sample, and the remaining utility function parameters are assumed to be transferable from the estimation context.
- This procedure works when the smaller amount of application context data can be sufficient to estimate scaling and constant parameters.

# Model Transfer Methods

## Bayesian Updating

- Parameter estimates from a small application context sample are combined with the estimation context parameter values estimated from a larger data set.
- The idea of Bayesian updating is to optimally combine the coefficients obtained from the application and estimation contexts, accounting for the variances (i.e., precision) of the coefficient estimates in the two contexts.
- This is done by computing a weighted average of the coefficients from the two contexts, the weights being equal to the inverse of the variance of the coefficient estimates.
- Bayesian updating is based on a combination approach that attaches more weight to more precise estimates.

# Model Transfer Methods

## Combined Transfer

- Generalization of the Bayesian updating process that includes transfer scaling (i.e., it combines the previous two methods).
- Allows difference in the model parameters in the populations.
- Updated parameters obtained as the minimum mean squared error estimate of the original and updated parameters.

## Model Transfer Methods

### *Joint Context Estimation*

- Involves the estimation of a single model using data from both the estimation and application contexts.
- Requires not only the model parameters from the estimation context, but the original estimation data set as well.
- One or more variable coefficients may be the same across the estimation and application areas; so there is a gain in efficiency in using data from both contexts.

# Summary of Procedures to Obtain Model Parameters

	Simple Transfer	Transfer Scaling	Bayesian Updating	Combined Transfer	Joint Context Estimation	Estimation of Parameters from Local Data
Required Information from Estimation	Model structure, parameter estimates	Model structure, parameter estimates	Model structure, parameter estimates and variances	Model structure, parameter estimates and variances	Complete model estimation data set	(not applicable)
Application Context Data Required (Survey)	None	Small sample	Small sample	Small sample	Small sample	Complete survey data set
Level of Expertise Required	Low	Medium	Medium	Medium	High	High
Level of Effort	Low	Low/ Medium	Medium	Medium/ High	High	High
Expected Transferability	Low	Low/ Medium	Medium	Medium/ High	Medium/ High	High

# Example of Model Transfer

## *Example Setting*

- Assume that “Urban Area A” wants to develop a cross-class HBW trip production model
- Assume any household survey data for Urban Area A are inadequate for estimating this model
- It is impossible to conduct a survey with sufficient data
- So Urban Area A is considering transferring a model from elsewhere
- “Urban Area B,” an area of similar size in the same state as Urban Area A, has conducted a household survey

# Example of Model Transfer

## *Example Setting*

- Assume that “Urban Area A” wants to develop a cross-classification trip production model for HBW trips
- Assume any household survey data for Urban Area A are inadequate for estimating this model
- It is it impossible to conduct a survey with sufficient data
- So Urban Area A is considering transferring a model from elsewhere.

# HBW Trip Productions

## *Urban Area B*

	Workers			
<b>Autos</b>	1	2	3+	<i>Average</i>
<b>0</b>	1.0	2.4	5.1	<b>0.7</b>
<b>1</b>	1.0	2.6	5.1	<b>0.9</b>
<b>2</b>	1.3	2.6	5.1	<b>2.0</b>
<b>3+</b>	1.3	2.6	5.1	<b>2.6</b>
<b>Average</b>	<b>1.1</b>	<b>2.6</b>	<b>5.1</b>	<b>1.5</b>



## Survey sample Sizes *Urban Area B*

	Workers				
<b>Autos</b>	0	1	2	3+	<i>Total</i>
<b>0</b>	500	300	100	25	<b>925</b>
<b>1</b>	1,000	1,500	400	25	<b>2,925</b>
<b>2</b>	300	1,000	1,500	100	<b>2,900</b>
<b>3+</b>	50	200	350	150	<b>750</b>
<b>Total</b>	<b>1,850</b>	<b>3,000</b>	<b>2,350</b>	<b>300</b>	<b>7,500</b>

## HBW Trip Productions

*Mean HBW Trip Rate Variances from Survey for Urban Area B*

	Workers		
Autos	1	2	3+
0	2.00	4.00	5.00
1	0.10	0.05	0.20
2	0.20	0.01	0.05
3+	0.30	0.02	0.04

# Simple Transfer

## *Use Rates from Urban Area B*

	Workers			
Autos	1	2	3+	<i>Average</i>
0	1.0	2.4	5.1	<b>0.7</b>
1	1.0	2.6	5.1	<b>0.9</b>
2	1.3	2.6	5.1	<b>2.0</b>
3+	1.3	2.6	5.1	<b>2.6</b>
<i>Average</i>	<b>1.1</b>	<b>2.6</b>	<b>5.1</b>	<b>1.5</b>

# Transfer Scaling

## *Use Rates from Urban Area B*

Assume that Urban Area A is able to obtain a small household survey data set for its region, with an average of 1.6 HBW trips/household

	Workers			
<b>Autos</b>	1	2	3+	<i>Average</i>
<b>0</b>	1.1	2.6	5.6	<b>0.8</b>
<b>1</b>	1.1	2.8	5.6	<b>1.0</b>
<b>2</b>	1.4	2.8	5.6	<b>2.2</b>
<b>3+</b>	1.4	2.8	5.6	<b>2.8</b>
<b>Average</b>	<b>1.2</b>	<b>2.8</b>	<b>5.6</b>	<b>1.6</b>

# Bayesian Updating

## *Sample Sizes for Small Survey for Urban Area A*

	Workers				
<b>Autos</b>	0	1	2	3+	<i>Total</i>
<b>0</b>	50	30	10	5	<b>95</b>
<b>1</b>	80	140	50	5	<b>275</b>
<b>2</b>	25	110	150	10	<b>295</b>
<b>3+</b>	10	25	30	20	<b>85</b>
<b>Total</b>	<b>165</b>	<b>305</b>	<b>240</b>	<b>40</b>	<b>750</b>

## Bayesian Updating

### *HBW Trip Productions Estimated from Small Survey for Urban Area A*

	Workers			
<b>Autos</b>	1	2	3+	<i>Average</i>
<b>0</b>	1.0	2.2	2.8	<b>0.7</b>
<b>1</b>	1.1	2.5	5.0	<b>1.1</b>
<b>2</b>	1.3	2.7	5.2	<b>2.0</b>
<b>3+</b>	1.4	3.0	5.2	<b>2.7</b>
<i>Average</i>	<b>1.2</b>	<b>2.7</b>	<b>4.9</b>	<b>1.6</b>

# Bayesian Updating

## *Mean HBW Trip Rate Variances from Small Survey for Urban Area A*

	Workers		
Autos	1	2	3+
0	5.00	25.0	50.0
1	2.00	1.00	1.20
2	5.00	0.50	2.00
3+	10.0	1.00	1.00

# Bayesian Updating

$$\hat{\theta}_{updated} = \frac{\left[ \frac{\hat{\theta}_{prior}}{(\hat{\sigma}_{prior}^2)} + \frac{\hat{\theta}_{updating}}{(\hat{\sigma}_{updating}^2)} \right]}{\left[ \frac{1}{(\hat{\sigma}_{prior}^2)} + \frac{1}{(\hat{\sigma}_{updating}^2)} \right]}$$

For example, for the 0 auto-1 worker, combination, the trip rate computation is:

$$[(1.0/2.00) + (1.2/5.00)] / [(1/2.00) + 1/5.00] = 1.06$$



# Bayesian Updating

$$\hat{\theta}_{updated} = \frac{\left[ \frac{\hat{\theta}_{prior}}{(\hat{\sigma}_{prior}^2)} + \frac{\hat{\theta}_{updating}}{(\hat{\sigma}_{updating}^2)} \right]}{\left[ \frac{1}{(\hat{\sigma}_{prior}^2)} + \frac{1}{(\hat{\sigma}_{updating}^2)} \right]}$$

For example, for the 0 auto-1 worker, combination, the trip rate computation is:

$$[(1.0/2.00) + (1.2/5.00)] / [(1/2.00) + 1/5.00] = 1.06$$

# Bayesian Updating

$$\hat{\theta}_{updated} = \frac{\left[ \frac{\hat{\theta}_{prior}}{(\hat{\sigma}_{prior}^2)} + \frac{\hat{\theta}_{updating}}{(\hat{\sigma}_{updating}^2)} \right]}{\left[ \frac{1}{(\hat{\sigma}_{prior}^2)} + \frac{1}{(\hat{\sigma}_{updating}^2)} \right]}$$

For example, for the 0 auto-1 worker, combination, the trip rate computation is:

$$[(1.0/2.00) + (1.2/5.00)] / [(1/2.00) + 1/5.00] = 1.06$$

# Example of Model Transfer

## *Combined Transfer*

- The Bayesian updating procedure assumes that the behavioral model parameters in the populations of urban areas A and B are identical.
- Thus, the procedure essentially obtains updated parameters for the urban area A (application area) as the weighted average of the estimates of  $\theta_{\text{prior}}$  and  $\theta_{\text{updating}}$ , where the weights correspond to the inverse of the variances of the estimated parameters.
- But there may be a real difference in the model parameters in the populations (sometimes labeled as transfer bias).
- In this case, one may obtain updated parameters as the minimum mean squared error estimate of  $\theta_{\text{prior}}$  and  $\theta_{\text{updating}}$

## Example of Model Transfer

### *Combined Transfer*

In the one-dimensional case presented here, the equivalent of the Bayesian updating equation in the combined transfer approach is as follows:

$$\hat{\theta}_{updated} = \frac{\left[ \frac{\hat{\theta}_{prior}}{(\hat{\sigma}_{prior}^2 + \hat{\Delta}^2)} + \frac{\hat{\theta}_{updating}}{(\hat{\sigma}_{updating}^2)} \right]}{\left[ \frac{1}{(\hat{\sigma}_{prior}^2 + \hat{\Delta}^2)} + \frac{1}{(\hat{\sigma}_{updating}^2)} \right]},$$

$$\hat{\Delta} = (\hat{\theta}_{updating} - \hat{\theta}_{prior})$$

# Example of Model Transfer

## *Combined Transfer*

- For example, for the 2 auto-3+ worker combination, the computation is:  
$$[(5.1/(0.05+0.1*0.1)) + (5.2/2.00)] / [(1/(0.05+0.1*0.1)) + (1/2.00)] = 5.1$$
- In this specific computation, the Bayesian and combined transfer approaches provide the same result up to the first decimal point.
- This is because of the substantial precision of  $\hat{\theta}_{prior}$ , but also because of the closeness of the estimates of  $\hat{\theta}_{prior}$  and  $\hat{\theta}_{updating}$ .
- In general, the combined transfer and Bayesian updating procedures will not provide the same estimates, except in the special case when  $\hat{\Delta} = 0$ .

# Example of Model Transfer

## *Joint Context Estimation Discussion*

- Joint context estimation generally requires individual-level disaggregate data from both the estimation and application contexts though the amount of data for the application context can be lesser.
- The basis of joint context estimation is that one or more variable coefficients may be the same across the estimation and application areas, and so there is a gain in efficiency in using data from both contexts.
- When data on a suite of independent variables are available from both the estimation and application contexts, the analyst can statistically test for which coefficient effects may be constrained to be the same, and which to let free.

# Example of Model Transfer

## *Joint Context Estimation Discussion*

$$\text{Trips} = B_1 (D_{00}) + B_2 (D_{10}) + B_3 (D_{01}) + B_4 (D_{11})$$

Where:

$v$  = number of autos (0 or 1+)

$w$  = number of workers (0 or 1+)

Trips = Total trips

$D_{vw} = 1$  if there are  $v$  autos and  $w$  workers, 0 otherwise

$B_k =$  Estimated coefficients, i.e., the trip rates

- The joint estimation would bring the household level observations from both the estimation and application contexts together, with an assumption that the overall magnitude of the effects of unobserved independent variables on trip production rates is the same in the estimation and application contexts.
- Then, a linear regression model set-up is developed, this time using the combined data set from both the estimation and application contexts.

# Example of Model Transfer

## *Joint Context Estimation Discussion*

$$\text{Trips} = B_1 (D_{00}) + B_2 (D_{10}) + B_3 (D_{01}) + B_4 (D_{11}) + B_5 (X_{00}) + B_6 (X_{10}) + B_7 (X_{01}) + B_8 (X_{11})$$

Where:

$v =$  number of autos (0 or 1+)

$w =$  number of workers (0 or 1+)

Trips = Total trips

$D_{vw} = 1$  if there are  $v$  autos and  $w$  workers, 0 otherwise

$E = 1$  if the observation belongs to the estimation context, 0 otherwise

$X_{vw} = D_{vw} * E$

$B_k =$  Estimated coefficients

- So the trip rates for the estimation area are given by:
  - 0 autos, 0 workers:  $B_5 + B_1$
  - 0 autos, 1+ workers:  $B_6 + B_2$
  - 1+ autos, 0 workers:  $B_7 + B_3$
  - 1+ autos, 1+ workers:  $B_8 + B_4$
- Now suppose that  $B_5$  is statistically insignificant
  - $X_{00}$  can be dropped from the linear regression, and one cannot reject the hypothesis that the expected value of trip production rate for 0 auto-0 worker households is the same between the estimation and application contexts (i.e.,  $B_5=B_1$ ). One can also relax the assumption of equal error variance if at least one coefficient is constrained between the estimation and application contexts.

This demonstrates the gain in efficiency from joint context estimation, as only those application context parameters that are statistically different from the estimation context parameters need be estimated, and joint context estimation provides a test for statistical significance of these differences.



# Traditional Model Transfer

What are the problems?

- It defines *a priori* what the similarity measures are and limits the dimensionality of similarity.
- It assumes a single uniform set of similarity measures regardless of the type of model being borrowed.
- The transfer is based on centralized measures of tendency between the two regions eliminating intrinsic heterogeneity within the two contexts.

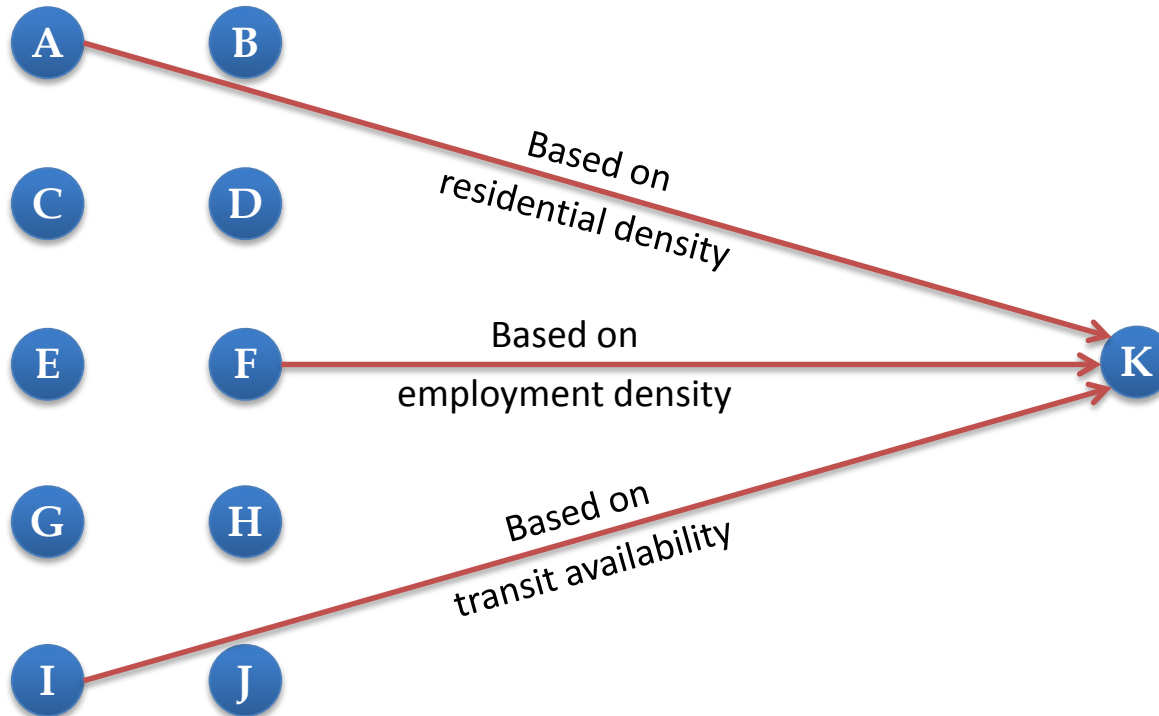
## New Approach

- This approach uses all available data from regions with information on the relevant activity-travel dimension of interest → one large estimation dataset.
- The estimation identifies a finite number of location similarity segments.

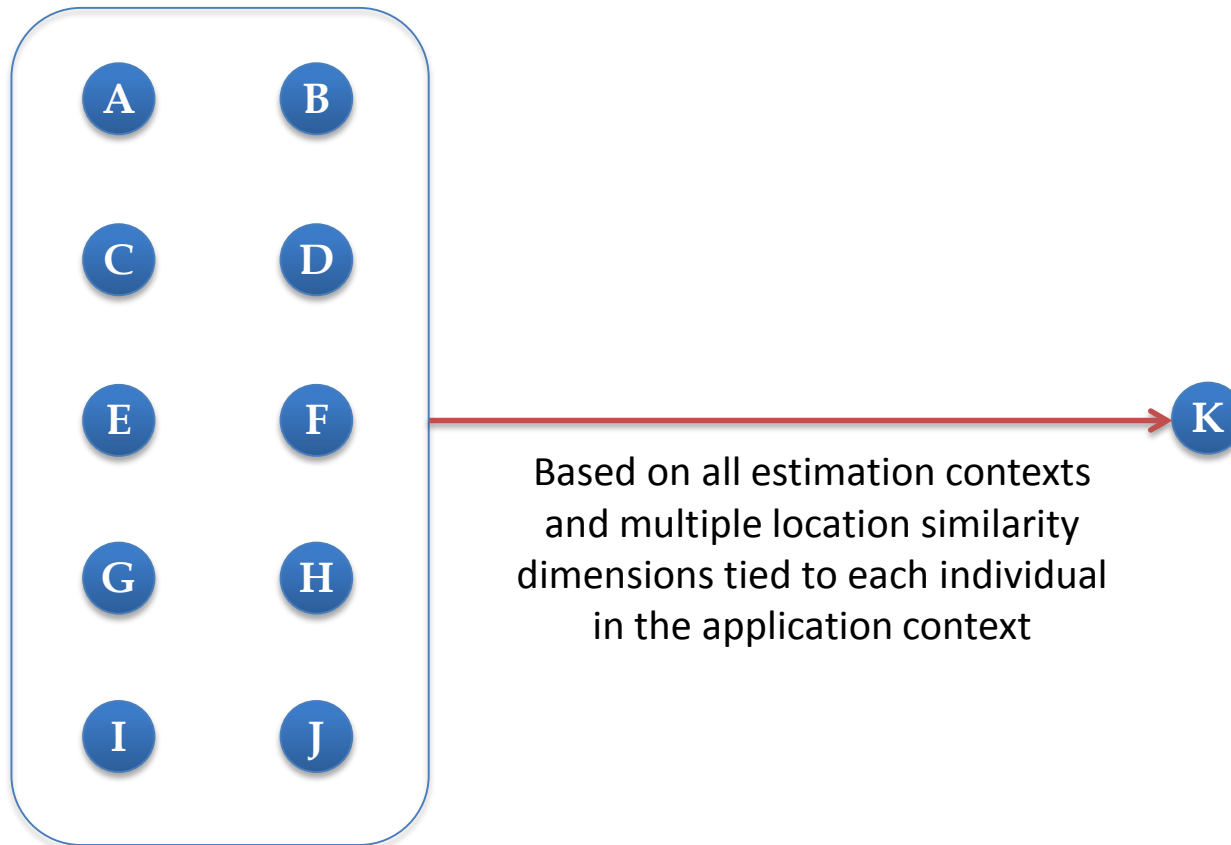
# Traditional Transfer Approach

Estimation Contexts

Application Context



# Proposed Transfer Approach



## Application

- Data for weekday travel of unemployed adults in the States of California and Florida (2009 NHTS).
- MDCEV model employed to simulate activity selection and participation.
- Three location segments, each with its own unique MDCEV parameter vector.
- Residential density, employment density, transit service quality found to be measures of location similarity.

# TMIP Updates

For future webinar announcement, please sign up for GovDelivery at <http://www.fhwa.dot.gov/planning/tmip/> if you have not done so.

# TMIP Contacts

If you have any questions or comments about today's presentation or TMIP, or if you are interested in sharing your experience, please contact me at:

[sarah.sun@dot.gov](mailto:sarah.sun@dot.gov) or  
[feedback@tmip.org](mailto:feedback@tmip.org).